Solution by the PROPOSER.

We have given $a_1^n+a_2^n+a_3^n+\ldots+a_r^n=A^n\ldots(1)$.

Dividing by A^n, $\left(\frac{a_1}{A}\right)^n+\left(\frac{a_2}{A}\right)^n+\left(\frac{a_3}{A}\right)^n+\ldots\left(\frac{a_r}{A}\right)^n=1\ldots(2)$.

Since the sum of the terms in the left member of (2) is 1, each term is <1. Hence each of the fractions

$$\frac{a_1}{A},\quad \frac{a_2}{A},\quad \frac{a_3}{A},\ \ldots,\ \frac{a_r}{A}$$

is a proper fraction. Then, if in (2) we substitute m for n, we shall have

$$\left(\frac{a_1}{A}\right)^m+\left(\frac{a_2}{A}\right)^m+\left(\frac{a_3}{A}\right)^m+\ldots+\left(\frac{a_r}{A}\right)^m> \text{ or } <1,$$

according as $m<$ or $>n$, as is clearly evident. Multiplying by A^m, $a_1{}^m+a_2{}^m+a_3{}^m+\ldots+a_r{}^m>$ or $<A^m$, according as $m<$ or $>n$.

271. Proposed by GEORGE H. HALLETT, Ph. D., Assistant Professor of Mathematics in The University of Pennsylvania, Philadelphia, Pa.

Find the simplest integral form of the sum $y(y-1)\ldots(y-x)+2y(2y-1)\ldots(2y-x)+\ldots+zy(zy-1)\ldots(zy-x)$.

Dr. Zerr obtains $\frac{1}{x!}\sum\limits_{r=y}^{r=zy}\int_0^1\left[\log\left(\frac{1}{u}\right)\right]^r du$ as the sum of the series. This does not satisfy the requirements of the problem for the reason that the sum is to be in integral form. ED. F.

272. Proposed by PROF. R. D. CARMICHAEL, Anniston, Ala.

Prove that the relations $x=\frac{ar+bs}{\lambda}=\frac{as-br}{\mu}=\frac{a\lambda-b\mu}{r}=\frac{a\mu+b\lambda}{s}$ between the finite real quantities x, a, b, r, s, λ, μ requires that $x^2=a^2+b^2$.

I. Solution by the PROPOSER.

These relations make the determinant

$$\triangle\equiv\begin{vmatrix}\lambda x+i\mu x & (ar+bs)+i(as-br)\\ (a\lambda-b\mu)+i(a\mu+b\lambda) & rx+isx\end{vmatrix},\ (i=\sqrt{-1}),$$

necessarily $=0$; for its columns are identical. Dividing the first column by $\lambda+i\mu$ and the second by $r+is$, we have

$$\triangle\equiv(\lambda+i\mu)(r+is)\begin{vmatrix}x & a-ib\\ a+ib & x\end{vmatrix}=0.$$

Hence,

$$\begin{vmatrix}x & a-ib\\ a+ib & x\end{vmatrix}=0;\ \text{or } x^2=a^2+b^2.$$